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# Some new integral relation of I- function

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## Abstract

This paper deals with some new integral relation of I- function of one variable.

## Introduction

The I- function of one variable is defined by Saxena [1] and we shall represent here in the following manner:

$$I[z] \quad = \quad \mathrm{I}_{p_i,q_i:r}^{m,n}\left[ z \left| \begin{matrix} [(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1},_{p_i}] \\ [(b_j,\beta_j)_{1,m}],[(b_{ji},\beta_{ji})_{m+1},_{q_i}] \end{matrix} \right. \right] \tag{1.1}$$

$$= \frac{1}{2\pi w}\int_L \theta(s) z^s ds,$$

where $\omega = \sqrt{(-1)}, z(\neq 0)$ is a complex variable and

$$z^s = \exp[s\{\log|z| + w \arg z\}]. \tag{1.2}$$

In which log |z| represent the natural logarithm of |z| and arg |z| is not necessarily the principle value. An empty product is interpreted as unity, also,

$$\theta(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j - \beta_j s)\prod_{j=1}^{n}\Gamma(1 - a_j - \alpha_j s)}{\sum_{i=1}^{r}\left[\prod_{j=m+1}^{q_i}\Gamma(1 - b_{ji} - \beta_{ji} s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji} - \alpha_{ji} s)\right]}, \tag{1.3}$$

m, n and pi $\forall$ i $\in$ (1,.....r) are non-negative integers satisfying 0 < n < $p_i$, 0 < m < $q_i$, $\forall$ i $\in$ (1,.....r), $\alpha_{ji}$, (j=1,..... $p_i$; i = 1,.........r) and $\beta_{ji}$ (j = 1,....$q_i$ ; i = 1,....r) are assumed to be positive quantities for standardization purpose . Also $a_{ji}$ ( j = 1,...., $p_i$; i = 1,......,r) and $b_{ji}$ ( j = 1,........, $q_i$; i = 1,.....,r) are complex numbers such that none of the points.

$$S = \{(bn + v) \mid \beta_h \mid\}, h = 1,......., m; v = 0,1,2,..... \tag{1.4}$$

which are the poles of $\Gamma$ ($b_n$ - $\beta_n$S), h = 1,......m and the points.

$$\mathrm{S} = \{(a_l - \eta - 1) \mid \alpha_l \mid l = 1,......, n; \eta = 0,1,2,....., \tag{1.5}$$

which are the poles of $\Gamma(1 - a_l + \alpha_l s)$ coincide with one another, i.e. with

$$\alpha_l(b_n + v) \quad \neq \quad b_n(a_l - \eta - 1) \tag{1.6}$$

for n, h = 0, 1,2,....; h = 1,...., m; l = 1,.....,n.

Further, the contour L runs from $-\omega\infty$ to $+\omega\infty$. Such that the poles of $\Gamma(b_n - \beta_n s)$, h = 1......, m; lie to the right of L and the poles $\Gamma(1 - a_l + \alpha_l s)$, l = 1,....,n lie to the left of L. The integral (1.1) converges, if |arg z| < ½ B π (B>0), A < 0, where

$$A = \sum_{j=1}^{p_i} a_{ji} - \sum_{j=1}^{q_i} \beta_{ji}. \tag{1.7}$$

and

$$B = \sum_{j=1}^{n} \alpha_j - \sum_{j=n+1}^{p_i} \alpha_{ji} + \sum_{j=1}^{m} \beta_j - \sum_{j=m+1}^{q_i} \beta_{ji}, \tag{1.8}$$

$\forall$ i $\in$ (1,.....,r).

Gradshteyin and Ryzhik [2] given table of Integrals, series, Sharma [3] evaluated the integrals involving general class of polynomial with H-function, Srivastava and Garg [4] established some integrals involving a general class of polynomials and the multivariable H-function. Recently, Satyanarayana and Pragathi Kumar [5] has evaluated Some finite integrals involving multivariable polynomials, Agarwal [6] established integral involving the product of Srivastava's polynomials and generalized Mellin-Barnes type of contour integral, Bhattar [7] established some integral formulas involving two $\overline{H}$ - function and multivariable's general class of polynomiyals. Satyanarayana and Pragathi Kumar [5] has evaluated some finite integrals involving multivariable polynomials. Following them, I evaluated some new integrals involving multivariable polynomials, and I-function of one variable.

## Formula Required

The following formulas will be required in our investigation

(i) The second class of multivariable polynomials given by Srivastava [8,9] is defined as follows:

$$S_{V_1,...,V_t}^{U_1,...,U_t}[x_1,...,x_t] = \sum_{K_1=0}^{[V_1/U_1]}...\sum_{K_t=0}^{[V_t/U_t]}(-V_1)_{U_1 k_1}...(-V_t)_{U_t k_t}\ \mathrm{A}[V_1,k_1;....;V_t,k_t]\ \frac{x_1^{k_1}}{k_1!}...\frac{x_t^{k_t}}{k_t!}. \tag{2.1}$$

(ii) The first class of multivariable polynomials introduced by Srivastava and Garg [4] is defined as follows:

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$$S_V^{U_1,\ldots,U_t}[x_1,\ldots,x_t]=\sum_{K_1\ldots k_t=o}^{U_1k_1+\ldots U_tK_t\le V}(-V_1)_{U_1k_1}\ldots(-V_t)_{U_tk_t}\ \mathrm{A}[V_1,k_1;\ldots;V_t,k_t]\ \frac{x_1^{k_1}}{k_1!}\ldots\frac{x_t^{k_t}}{k_t!}$$

## Some New Finite Integrals Formulae

In this section we prove two integral formulae, which involving multivariable polynomials, and I function of one variable.

$$\int_{-1}^{1}(1-x)^{\rho}(1+x)^{\sigma}\ \mathrm{S}_{\mathrm{V}_1,\ldots,\mathrm{V}_t}^{U_1,\ldots,U_t}[\mathrm{y}_1(1-x)^{m_1}(1+x)^{n_1},\ldots,\mathrm{y}_t(1-x)^{m_t}(1+x)^{n_t}]$$

$$\times I_{p_i,q_i:r}^{m,n}\left[z(1-x)^{g}(1+x)^{h}\left|\begin{matrix}[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}]\\ [(b_j,\beta_j)_{1,n}],[(b_{ji},\beta_{ji})_{n+1,p_i}]\end{matrix}\right.\right]\mathrm{dx}$$

$$=2^{\rho+\sigma+1}\sum_{K_1=0}^{[\mathrm{V}_1/\mathrm{U}_1]}\ldots\ldots\sum_{K_t=0}^{[\mathrm{V}_t/\mathrm{U}_t]}(-V_1)_{U_1k_1}\ldots(-V_t)_{U_tK_t}\ \mathrm{A}[\mathrm{V}_1,k_1;\ldots;\mathrm{V}_t,k_t]\ \frac{y_1^{k_1}}{k_1!}\ldots\frac{y_t^{k_t}}{k_t!}\ 2^{\sum_{i=1}^{t}(m_i+n_i)\mathrm{k}_i}$$

$$\times I_{p_i,q_i:r}^{m,n}\left[z2^{h+g}\left|\begin{matrix}(-\sigma-\sum_{i=1}^{t}m_ik_i,h;1),(-\rho-\sum_{i=1}^{t}n_ik_i\ ,g;1)[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}]\\ [(b_j,\beta_j)_{1,n}],[(b_{ji},\beta_{ji})_{n+1,q_i}](-\sigma-\rho-\sum_{i=1}^{t}(m_i+n_i)k_i-1,h+g;1)\end{matrix}\right.\right]\quad(3.1)$$

where $m_i>0$ (i=1,…,t), $n_i>0$ (i = 1,….,t) $h\ge 0$ ,$g\ge 0$ (not both are zero simultaneously).

$$\int_{-1}^{1}(1-x)^{\rho}(1+x)^{\sigma}S_V^{U_1\ldots U_t}[y_1(1-x)^{m_1}(1+x)^{n_1},\ldots,y_t(1-x)^{m_t}(1+x)^{n_t}]$$

$$\times I_{p_i,q_i:r}^{m,n}\left[z(1-x)^{g}(1+x)^{h}\left|\begin{matrix}[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}]\\ [(b_j,\beta_j)_{1,n}],[(b_{ji},\beta_{ji})_{n+1,p_i}]\end{matrix}\right.\right]\mathrm{dx}$$

$$=2^{\rho+\sigma+1}\sum_{k_1\ldots,k_t=0}^{U_1k_1+\ldots+U_tk_t}(-V_1)_{U_1k_1+\ldots+U_tk_t}A(V_1,k_1;\ldots;k_t)\frac{y_1^{k_1}}{k_1!}\ldots\frac{y_t^{k_t}}{k_t!}2^{\sum_{i=1}^{t}(m_i+n_i)k_i}$$

$$\times I_{p_i+2,\mathrm{q}_i+1:r}^{\mathrm{m,n+2}}\left[z2^{h+g}\left|\begin{matrix}(-\sigma-\sum_{i=1}^{t}m_ik_i,h;1),(-\rho-\sum_{i=1}^{t}n_ik_i,g;1),[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}]\\ [(b_j,\beta_j)_{1,\mathrm{m}}],[(b_{ji},\beta_{ji})_{m+1,q_i}],(-\sigma-\rho-\sum_{i=1}^{t}(m_i+n_i)k_i-1,h+g;1)\end{matrix}\right.\right],\quad(3.2)$$

Provided the conditions stated in results (3.1) are satisfied.

**Proof :** To establish integral in (3.1), we express I-function occurring in its left –hand side interms of Mellin-Barnes [10] contour integral given by (3.1), the second class of polynomial given by (2.1). Then interchange the order of integration of summations and integration, we arrive the following:

$$\sum_{k_1=0}^{[V_1/U_1]}\ldots\sum_{k_t=0}^{[V_t/U_t]}(-V_1)_{U_1k_1}\ldots(-V_t)_{U_tk_t}A(V_1,k_1;\ldots;V_t,k_t)\frac{y_1^{k_1}}{k_1!}\ldots\frac{y_t^{k_t}}{k_t!}$$

$$\mathrm{x}\ \ \frac{1}{2\pi i}\int_L\phi(s)z^s\int_{-1}^{1}(1-x)^{\rho+gs+\sum_{i=1}^{t}m_ik_i}(1+x)^{\sigma+hx+\sum_{i=1}^{t}n_ik_i}dxds$$

$$=\sum_{k_1=0}^{[V_1/U_1]}\ldots\sum_{k_t=0}^{[V_t/U_t]}(-V_1)_{U_1k_1}\ldots(-V_t)_{U_tk_t}A(V_1,k_1;\ldots;V_t,k_t)\frac{y_1^{k_1}}{k_1!}\ldots\frac{y_t^{k_t}}{k_t!}$$

$$\times\ \ \frac{1}{2\pi i}\int_L\phi(s)z^sds\ 2^{\sigma+hs+\sum_{i=1}^{t}n_ik_i+\rho+gs+\sum_{i=1}^{t}m_ik_i+1}\ \frac{\Gamma(\sigma+hs+\sum_{i=1}^{t}n_ik_i+1)\Gamma(\rho+gs+\sum_{i=1}^{t}m_ik_i+1)}{\Gamma(\sigma+hs+\sum_{i=1}^{t}n_ik_i+\rho+gs+\sum_{i=1}^{t}m_ik_i+2)}$$

$$=\ 2^{\sigma+\rho+1}\sum_{k_1=0}^{[V_1/U_1]}\ldots\sum_{k_t=0}^{[V_t/U_t]}(-V_1)_{U_1k_1}\ldots(-V_t)_{U_tk_t}A(V_1,k_1;\ldots;V_t,k_t)\frac{y_1^{k_1}}{k_1!}\ldots\frac{y_t^{k_t}}{k_t!}2^{\sum_{i=1}^{t}(m_i+n_i)k_i}$$

$$\times\ \ \frac{1}{2\pi i}\int_L\phi(s)z^s\frac{\Gamma(\sigma+hs+\sum_{i=1}^{t}n_ik_i+1)\Gamma(\rho+gs+\sum_{i=1}^{t}m_ik_i+1)}{\Gamma(\sigma+hs+\sum_{i=1}^{t}n_ik_i+\rho+gs+\sum_{i=1}^{t}m_ik_i+2)}$$

$$(z2^{h+g})^s\,ds\,\cdot$$

Now we evaluate the above integral with help of integral (2.2). Interpreting the resulting contour integral of H-function we can easily arrive at desired result (3.1).

To establish integral in (3.2) can be easily established on the same lines similar to the proof of (3.1).

## Special Cases of (3.1) and (3.2)

Take A $(V_1,k_1;\ldots;V_t,k_t)$ = $A_1(V_1,k_1)\ldots\ A_t\ (V_t,k_t)$ in (3.1) the multivariable polynomial $S_{V_1\ldots V_t}^{U_1\ldots U_t}(x_1,\ldots,x_t)$ reduced to the product of well-known general class of polynomials $S_v^U(x)$ and the result (3.1) reduced to following form

$$\int_{-1}^{1}(1-x)^{\rho}(1+x)^{\sigma}\prod_{i=1}^{t}S_{V_i}^{U_i}[y_i(1-x)^m(1+x)^n]$$

$$\times I_{p_i,q_i:r}^{m,n}\left[z(1-x)^{g}(1+x)^{h}\left|\begin{matrix}[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}]\\ [(b_j,\beta_j)_{1,n}],[(b_{ji},\beta_{ji})_{n+1,\mathrm{q}_i}]\end{matrix}\right.\right]\mathrm{dx}$$

$$2^{\sigma+\rho+1}\sum_{k_1=0}^{[V_1/U_1]}\ldots\sum_{k_t=0}^{[V_t/U_t]}(-V_1)_{U_1k_1}\ldots(-V_t)_{U_tk_t}A(V_1,k_1\ldots,V_tk_t)\frac{y_1^{k_1}}{k_1!}\ldots\frac{y_t^{k_t}}{k_t!}2^{\sum_{i=1}^{t}(m_i+n_i)k_i}$$

$$\times I_{p_i+2,\mathrm{q}_i+1:r}^{\mathrm{m,n+2}}\left[z2^{h+g}\left|\begin{matrix}(-\sigma-\sum_{i=1}^{t}m_ik_i,h,1),(-\rho-\sum_{i=1}^{t}n_ik_i,g;1)[(a_j,\alpha_j)_{1,\mathrm{n}}],[(a_{ji},\alpha_{ji})_{n+1,\mathrm{p}_i}]\\ [(b_j,\beta_j)_{1,\mathrm{m}}],[(b_{ji},\beta_{ji})_{m+1,q_i}],(-\sigma-\rho-\sum_{i=1}^{t}(m_i+n_i)k_i-1,h+g;1)\end{matrix}\right.\right]\quad(4.1)$$

**(a)Substituting r=1 in (3.1), we obtain :**

$$\int_{-1}^{1}(1-x)^{\rho}(1+x)^{\sigma}\ \mathrm{S}_{\mathrm{V}_1,\ldots,\mathrm{V}_t}^{U_1,\ldots,U_t}[\mathrm{y}_1(1-x)^{m_1}(1+x)^{n_1},\ldots,\mathrm{y}_t(1-x)^{m_t}(1+x)^{n_t}]$$

$$\times\ \mathrm{H}_{p,q}^{m,n}\left[z(1-x)^{g}(1+x)^{h}\left|\begin{matrix}(a_j,\alpha_j)_{1,p}\\ (b_j,\beta_j)_{1,q}\end{matrix}\right.\right]\mathrm{dx}$$

$$=2^{\rho+\sigma+1}\sum_{K_1=0}^{[\mathrm{V}_1/\mathrm{U}_1]}\ldots\ldots\sum_{K_t=0}^{[\mathrm{V}_t/\mathrm{U}_t]}\frac{(-V_1)_{U_1k_1}}{k_1!}\ldots\frac{(-V_t)_{U_tK_t}}{k_t!}\ \mathrm{A}[\mathrm{V}_1,k_1;\ldots;\mathrm{V}_t,k_t]\ \frac{y_1^{k_1}}{k_1!}\ldots\frac{y_t^{k_t}}{k_t!}\ \sum_{i=1}^{t}(m_i+n_i)\,\mathrm{k}_i$$

$$\times\ \mathrm{H}_{p,q}^{m,n}\left[z2^{h+g}\left|\begin{matrix}(-\sigma-\sum_{i=1}^{t}m_ik_i,h;1),(-\rho-\sum_{i=1}^{t}n_ik_i\ ,g;1)(a_j,\alpha_j)_{1,\mathrm{p}}\\ (b_j,\beta_j)_{1,\mathrm{q}}(-\sigma-\rho-\sum_{i=1}^{t}(m_i+n_i)k_i-1,h+g;1)\end{matrix}\right.\right]\quad(4.2)$$

**(b)Substituting $\alpha_j=\beta_j=1$ in (4.2) we obtain**

$$\int_{-1}^{1}(1-x)^{\rho}(1+x)^{\sigma}S_{V_1\ldots V_t}^{U_1\ldots U_t}[y_1(1-x)^{m_1}(1+x)^{n_1}\ldots y_t(1-x)^{m_t}(1+x)^{n_t}]$$

$$\times\ G_{\mathrm{p,q}}^{\mathrm{m,n}}\left[z(1-x)^{g}(1+x)^{h}\left|\begin{matrix}(a_j)_{1,\mathrm{p}}\\ (b_j)_{1,\mathrm{q}}\end{matrix}\right.\right]dx$$

$$2^{\sigma+\rho+1}\sum_{k_1=0}^{[V_1/U_1]}\ldots\sum_{k_t=0}^{[V_t/U_t]}(-V_1)_{U_1k_1}\ldots(-V_t)_{U_tk_t}A(V_1,k_1\ldots,V_t,k_t)\frac{y_1^{k_1}}{k_1!}\ldots\frac{y_t^{k_t}}{k_t!}2^{\sum_{i=1}^{t}(m_i+n_i)k_i}$$

$$\times G_{p+2,\mathrm{q}+1}^{\mathrm{m,n+2}}\left[z2^{h+g}\left|\begin{matrix}(-\sigma-\sum_{i=1}^{t}m_ik_i,h,1),(-\rho-\sum_{i=1}^{t}n_ik_i,g;1)(a_j)_{1,\mathrm{p}}\\ (b_j)_{1,\mathrm{q}},(-\sigma-\rho-\sum_{i=1}^{t}(m_i+n_i)k_i-1,h+g;1)\end{matrix}\right.\right].\quad(4.3)$$

(a)Substituting r=1 in (3.2), we obtain :

$$\int_{-1}^{1}(1-x)^{p}(1+x)^{q}S_V^{U_1\ldots U_t}[y_1(1-x)^m(1+x)^n,\ldots,y_t(1-x)^m(1+x)^n]$$

$$\times\ \mathrm{H}_{p,q}^{m,n}\left[z(1-x)^{g}(1+x)^{h}\left|\begin{matrix}(a_j,\alpha_j)_{1,\mathrm{p}}\\ (b_j,\beta_j)_{1,\mathrm{q}}\end{matrix}\right.\right]\mathrm{dx}$$

$$= 2^{\rho+\sigma+1} \sum_{k_1\ldots,k_t=0}^{U_1k_1+\ldots+U_tk_t} (-V_1)_{U_1k_1+\ldots+Utkt} A(V_1,k_1;\ldots,k_t) \frac{y_1^{k_1}}{k_1!}\cdots\frac{y_t^{k_t}}{k_t!} 2^{\sum_{i=1}^{t}(m_i+n_i)k_i}$$

$$\times H^{m,n+2}_{p+2,q+1}\left[ z2^{h+g}\left|\begin{matrix}(-\sigma-\sum_{i=1}^{t} m_ik_i,h,1),(-\rho-\sum_{i=1}^{t} n_ik_i,g;1),(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q},(-\sigma-\rho-\sum_{i=1}^{t}(m_i+n_i)k_i-1,h+g;1)\end{matrix}\right.\right] \qquad (4.4)$$

(b)Substituting $\alpha_j=\beta_j=1$ in (4.4), we obtain

$$\int_{-1}^{1}(1-x)^{\rho}(1+x)^{\sigma} S_V^{U_1,\ldots,U_t}[y_1(1-x)^m(1+x)^n,\ldots,y_t(1-x)^m(1+x)^n]$$

$$\times\, G^{m,n}_{p,q}\left[z(1-x)^g(1+x)^h\left|\begin{matrix}(a_j)_{1,p}\\(b_j)_{1,q}\end{matrix}\right.\right]\, dx$$

$$= 2^{\rho+\sigma+1} \sum_{k_1\ldots,k_t=0}^{U_1k_1+\ldots+U_tk_t} (-V_1)_{U_1k_1+\ldots+U_tk_t} A(V_1,k_1;\ldots,k_t) \frac{y_1^{k_1}}{k_1!}\cdots\frac{y_t^{k_t}}{k_t!} 2^{\sum_{i=1}^{t}(m_i+n_i)k_i}$$

$$\times G^{m,n+2}_{p+2,q+1}\left[ z2^{h+g}\left|\begin{matrix}(-\sigma-\sum_{i=1}^{t} m_ik_i,h,1),(-\rho-\sum_{i=1}^{t} n_ik_i,g;1),(a_j)_{1,p}\\(b_j)_{1,q},(-\sigma-\rho-\sum_{i=1}^{t}(m_i+n_i)k_i-1,h+g;1)\end{matrix}\right.\right]$$

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